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New $N=(2,2)$ vector multiplets

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Abstract: We introduce two new $N=(2,2)$ vector multiplets that couple naturally to generalized Kähler geometries. We describe their kinetic actions as well as their matter couplings both in $N=(2,2)$ and $N=(1,1)$ superspace.

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1. Introduction

Generalized Kähler geometry has aroused considerable interest both among string theorists and mathematicians, e.g., [1 – 3]. Recently, several groups have tried to construct quotients [4 – 7]; however, it is unclear how general or useful the various proposals are. Experience has shown that supersymmetric σ-models are often a helpful guide to finding the correct geometric concepts and framework for quotient constructions [8, 9]. In this paper, we take the first step in this direction; further results will be presented in [10].

The basic inspiration for our work is the interesting duality found in [11, 12]. As was shown in [9, 13], T-dualities arise when one gauges an isometry, and then constrains the field-strength of the corresponding gauge multiplet to vanish. Here we address the question: what are the gauge multiplets corresponding to the duality introduced in [11, 12]?

In section 2, we analyze the types of isometries that arise on generalized Kähler geometries which are suitable for gauging, and describe the corresponding multiplets in $N=(2,2)$

superspace. In addition to the usual multiplets with chiral or twisted chiral gauge parameters, we find two new multiplets: one with semichiral gauge parameters, which we call the semichiral gauge multiplet, and one with a pair of gauge parameters, one chiral and one twisted chiral; the last has more gauge-invariant components than other multiplets, and hence we call it the large vector multiplet.

In section 3, we describe the $N = (1,1)$ superspace content of these mulitplets; this exposes their physical content. We describe both multiplets and their couplings to matter, and discuss possible gauge actions for them. The component content of the various $N = (1,1)$ multiplets that arise is well known.

Throughout this paper we follow the conventions of [14].

2. Generalized Kähler geometry: $N = (2,2)$ superspace

Generalized Kähler geometry (GKG) arises naturally as the target space of $N = (2,2)$ supersymmetric σ-models. As shown in [14], such σ-models always admit a local description in $N = (2,2)$ superspace in terms of complex chiral superfields ϕ, twisted chiral superfields χ and semichiral superfields $\mathbb{X}_L, \mathbb{X}_R$ [15]. These models have also been considered in $N = (1,1)$ superspace [16, 17].

These geometries may admit a variety of holomorphic isometries that can be gauged by different kinds of vector multiplets. We now itemize the basic types of isometries.

2.1 Isometries

The simplest isometries act on purely Kähler submanifolds of the generalized Kähler geometry, that is only on the chiral superfields ϕ or the twisted chiral superfields χ; for a single $\mathrm{U}(1)$ isometry away from a fixed point, we may choose coordinates so that the Killing vectors take the form:

$$k_\phi = i(\partial_\phi - \partial_{\bar\phi}) \ , \qquad k_\chi = i(\partial_\chi - \partial_{\bar\chi}) \ . \tag{2.1}$$

In [11, 12], new isometries that mix chiral and twisted chiral superfields or act on semichiral superfields were discovered; we may take them to act as

$$k_{\phi\chi} = i(\partial_\phi - \partial_{\bar\phi} - \partial_\chi + \partial_{\bar\chi}) \, , \tag{2.2}$$

$$k_{LR} = i(\partial_L - \partial_{\bar L} - \partial_R + \partial_{\bar R}) \, , \tag{2.3}$$

where $\partial_L = \frac{\partial}{\partial \mathbb{X}_L}$, etc. One might imagine more general isometries that act along an arbitrary vector field; however, compatibility with the constraints on the superfields (chiral and twisted chiral superfields are automatically semichiral but not vice-versa) allows us to restrict to the cases above; in particular, if the vector field has a component along k_ϕ, k_χ or $k_{\phi\chi}$, we can (locally) redefine $\mathbb{X}$ to eliminate any component along k_{LR}.

A general Lagrange density in $N = (2,2)$ superspace has the form:

$$K = K(\phi, \bar\phi, \chi, \bar\chi, \mathbb{X}_L, \bar{\mathbb{X}}_L, \mathbb{X}_R, \bar{\mathbb{X}}_R) \tag{2.4}$$

For the four isometries listed above the corresponding invariant Lagrange densities are:[1]

$$k_\phi\, K(\phi+\bar\phi,\chi,\bar\chi,\mathbb{X}_L,\bar{\mathbb{X}}_L,\mathbb{X}_R,\bar{\mathbb{X}}_R) = 0 \tag{2.5}$$
$$k_\chi\, K(\phi,\bar\phi,\chi+\bar\chi,\mathbb{X}_L,\bar{\mathbb{X}}_L,\mathbb{X}_R,\bar{\mathbb{X}}_R) = 0 \tag{2.6}$$
$$k_{\phi\chi}\, K(\phi+\bar\phi,\chi+\bar\chi,i(\phi-\bar\phi+\chi-\bar\chi),\mathbb{X}_L,\bar{\mathbb{X}}_L,\mathbb{X}_R,\bar{\mathbb{X}}_R) = 0 \tag{2.7}$$
$$k_{LR}\, K(\phi,\bar\phi,\chi,\bar\chi,\mathbb{X}_L+\bar{\mathbb{X}}_L,\mathbb{X}_R+\bar{\mathbb{X}}_R,i(\mathbb{X}_L-\bar{\mathbb{X}}_L+\mathbb{X}_R-\bar{\mathbb{X}}_R)) = 0 \tag{2.8}$$

In general, the isometries act on the coordinates with some constant parameter λ:

$$\delta z = [\lambda k, z] \ , \tag{2.9}$$

where z is any of the coordinates $\phi, \chi, \mathbb{X}_L, \mathbb{X}_R$, etc.

2.2 Gauging and vector multiplets

We now promote the isometries to local gauge symmetries: the constant transformation parameter λ of (2.9) becomes a local parameter Λ that obeys the appropriate constraints.

$$\begin{aligned}
\delta_g\phi = i\Lambda &\Rightarrow \bar{\mathbb{D}}_\pm\Lambda = 0\\
\delta_g\bar\phi = -i\bar\Lambda &\Rightarrow \mathbb{D}_\pm\bar\Lambda = 0\\
\delta_g\chi = i\tilde\Lambda &\Rightarrow \bar{\mathbb{D}}_+\tilde\Lambda = \mathbb{D}_-\tilde\Lambda = 0\\
\delta_g\bar\chi = -i\bar{\tilde\Lambda} &\Rightarrow \mathbb{D}_+\bar{\tilde\Lambda} = \bar{\mathbb{D}}_-\bar{\tilde\Lambda} = 0\\
\delta_g\mathbb{X}_L = i\Lambda_L &\Rightarrow \bar{\mathbb{D}}_+\Lambda_L = 0\\
\delta_g\mathbb{X}_R = i\Lambda_R &\Rightarrow \bar{\mathbb{D}}_-\Lambda_R = 0\\
\delta_g\bar{\mathbb{X}}_L = -i\bar\Lambda_L &\Rightarrow \mathbb{D}_+\bar\Lambda_L = 0\\
\delta_g\bar{\mathbb{X}}_R = -i\Lambda_R &\Rightarrow \mathbb{D}_-\bar\Lambda_R = 0\ .
\end{aligned} \tag{2.10}$$

To ensure the invariance of the Lagrange densities (2.5), (2.8) under the local transformations (2.10), we introduce the appropriate vector multiplets. For the isometries (2.5), (2.6) these give the well known transformation properties for the usual (un)twisted vector multiplets:

$$\begin{aligned}
\delta_g V^\phi = i(\bar\Lambda-\Lambda) &\Rightarrow \delta_g(\phi+\bar\phi+V^\phi) = 0\\
\delta_g V^\chi = i(\bar{\tilde\Lambda}-\tilde\Lambda) &\Rightarrow \delta_g(\chi+\bar\chi+V^\chi) = 0\,,
\end{aligned} \tag{2.11}$$

whereas for generalized Kähler transformations we need to add *triplets* of vector multiplets.

For the the semichiral isometry k_{LR}, we introduce the vector multiplets:

$$\begin{aligned}
\delta_g\mathbb{V}^L = i(\bar\Lambda_L-\Lambda_L) &\Rightarrow \delta_g(\mathbb{X}_L+\bar{\mathbb{X}}_L+\mathbb{V}^L) = 0\\
\delta_g\mathbb{V}^R = i(\bar\Lambda_R-\Lambda_R) &\Rightarrow \delta_g(\mathbb{X}_R+\bar{\mathbb{X}}_R+\mathbb{V}^R) = 0\\
\delta_g\mathbb{V}' = \Lambda_L+\bar\Lambda_L+\Lambda_R+\bar\Lambda_R &\Rightarrow \delta_g(i(\mathbb{X}_L-\bar{\mathbb{X}}_L+\mathbb{X}_R-\bar{\mathbb{X}}_R)+\mathbb{V}') = 0\ .
\end{aligned} \tag{2.12}$$

[1] Generally, isometries may leave the Lagrange density invariant only up to a (generalized) Kähler transformation [19, 14], but as our interest here is the structure of the vector multiplet, we are free to choose the simplest situation.

We refer to this multiplet as the semichiral vector multiplet.

For the $k_{\phi\chi}$ isometry we introduce the vector multiplets

$$\begin{aligned}\delta_g V^\phi = i(\bar\Lambda - \Lambda) &\Rightarrow \delta_g(\phi + \bar\phi + V^\phi) = 0\\ \delta_g V^\chi = i(\bar{\tilde\Lambda} - \tilde\Lambda) &\Rightarrow \delta_g(\chi + \bar\chi + V^\chi) = 0\\ \delta_g V' = \Lambda + \bar\Lambda + \tilde\Lambda + \bar{\tilde\Lambda} &\Rightarrow \delta_g(i(\phi - \bar\phi + \chi - \bar\chi) + V') = 0\,,\end{aligned} \tag{2.13}$$

and refer to this multiplet as the large vector multiplet due to the large number of gauge-invariant components that comprise it.

2.3 $N=(2,2)$ field-strengths

We now construct the $N=(2,2)$ gauge invariant field-strengths for the various multiplets introduced above.

2.3.1 The known field-strengths

The field-strengths for the usual vector multiplets are well known:

$$\begin{aligned}\tilde W = i\,\mathbb{D}_-\bar{\mathbb{D}}_+ V^\phi\ , &\quad \bar{\tilde W} = i\,\bar{\mathbb{D}}_-\mathbb{D}_+ V^\phi\,,\\ W = i\,\bar{\mathbb{D}}_-\bar{\mathbb{D}}_+ V^\chi\ , &\quad \bar W = i\,\mathbb{D}_-\mathbb{D}_+ V^\chi\ .\end{aligned} \tag{2.14}$$

Note that $\tilde W$, the field-strength for the chiral isometry is twisted chiral whereas W, the field-strength for the twisted chiral isometry, is chiral.

2.3.2 Semichiral field-strengths

To find the gauge-invariant field-strengths for the vector multiplet that gauges the semichiral isometry it is useful to introduce the complex combinations:

$$\begin{aligned}\mathbb{V} &= \frac{1}{2}(\mathbb{V}' + i(\mathbb{V}^L + \mathbb{V}^R)) \Rightarrow \delta_g\mathbb{V} = \Lambda_L + \Lambda_R\,,\\ \tilde{\mathbb{V}} &= \frac{1}{2}(\mathbb{V}' + i(\mathbb{V}^L - \mathbb{V}^R)) \Rightarrow \delta_g\tilde{\mathbb{V}} = \Lambda_L + \bar\Lambda_R\ .\end{aligned} \tag{2.15}$$

Then the following complex field-strengths are gauge invariant:

$$\begin{aligned}\mathbb{F} = \bar{\mathbb{D}}_+\bar{\mathbb{D}}_-\mathbb{V}\ , &\quad \bar{\mathbb{F}} = -\mathbb{D}_+\mathbb{D}_-\bar{\mathbb{V}}\,,\\ \tilde{\mathbb{F}} = \bar{\mathbb{D}}_+\mathbb{D}_-\tilde{\mathbb{V}}\ , &\quad \bar{\tilde{\mathbb{F}}} = -\mathbb{D}_+\bar{\mathbb{D}}_-\bar{\tilde{\mathbb{V}}}\,,\end{aligned} \tag{2.16}$$

where $\mathbb{F}$ is chiral and $\tilde{\mathbb{F}}$ is twisted chiral.

2.3.3 Large vector multiplet field-strengths

As above it is useful to introduce the complex potentials:

$$\begin{aligned}V &= \frac{1}{2}[V' + i(V^\phi + V^\chi)] \Rightarrow \delta_g V = \Lambda + \tilde\Lambda\,,\\ \tilde V &= \frac{1}{2}[V' + i(V^\phi - V^\chi)] \Rightarrow \delta_g\tilde V = \Lambda + \bar{\tilde\Lambda}\ .\end{aligned} \tag{2.17}$$

Because $(\tilde{\Lambda})\Lambda$ are (twisted)chiral respectively, the following complex spinor field-strengths are gauge invariant:

$$\begin{aligned} \mathbb{G}_+ &= \bar{\mathbb{D}}_+ V \ , \quad \bar{\mathbb{G}}_+ = \mathbb{D}_+ \bar{V} \, , \\ \mathbb{G}_- &= \bar{\mathbb{D}}_- \tilde{V} \ , \quad \bar{\mathbb{G}}_- = \mathbb{D}_- \bar{\tilde{V}} \ . \end{aligned} \tag{2.18}$$

The higher dimension field-strengths can all be constructed from these spinor field-strengths:

$$\begin{aligned} W &= -i\bar{\mathbb{D}}_+\bar{\mathbb{D}}_- V^\chi = \bar{\mathbb{D}}_+\mathbb{G}_- + \bar{\mathbb{D}}_-\mathbb{G}_+ \\ \bar{W} &= -i\mathbb{D}_+\mathbb{D}_- V^\chi = -(\mathbb{D}_+\bar{\mathbb{G}}_- + \mathbb{D}_-\bar{\mathbb{G}}_+) \\ \tilde{W} &= -i\mathbb{D}_+\bar{\mathbb{D}}_- V^\phi = \bar{\mathbb{D}}_+\bar{\mathbb{G}}_- + \mathbb{D}_-\mathbb{G}_+ \\ \bar{\tilde{W}} &= -i\bar{\mathbb{D}}_+\mathbb{D}_- V^\phi = -(\mathbb{D}_+\mathbb{G}_- + \bar{\mathbb{D}}_-\bar{\mathbb{G}}_+) \\ B &= -\bar{\mathbb{D}}_+\bar{\mathbb{D}}_-(V' + iV^\phi) = \bar{\mathbb{D}}_-\mathbb{G}_+ - \bar{\mathbb{D}}_+\mathbb{G}_- \\ \bar{B} &= \mathbb{D}_+\mathbb{D}_-(V' - iV^\phi) = -(\mathbb{D}_-\bar{\mathbb{G}}_+ - \mathbb{D}_+\bar{\mathbb{G}}_-) \\ \tilde{B} &= -\mathbb{D}_+\bar{\mathbb{D}}_-(V' - iV^\chi) = \mathbb{D}_-\mathbb{G}_+ - \bar{\mathbb{D}}_+\bar{\mathbb{G}}_- \\ \bar{\tilde{B}} &= \bar{\mathbb{D}}_+\mathbb{D}_-(V' + iV^\chi) = -(\bar{\mathbb{D}}_-\bar{\mathbb{G}}_+ - \mathbb{D}_+\mathbb{G}_-) \ ; \end{aligned} \tag{2.19}$$

the chirality properties of these field-strengths are summarized below:

Field-strength	Property
W, B	chiral
$\bar{W}, \bar{B}$	anti-chiral
$\tilde{W}, \tilde{B}$	twisted chiral
$\bar{\tilde{W}}, \bar{\tilde{B}}$	anti-twisted chiral

(2.20)

3. Gauge multiplets in $N=(1,1)$ superspace

To reveal the physical content of the gauge multiplets, we could go to components, but it is simpler and more informative to go to $N=(1,1)$ superspace. We expect to find spinor gauge connections and unconstrained superfields.

The procedure for going to $N=(1,1)$ components is well-known; for a convenient review, see [14]. We write the $N=(2,2)$ derivatives $\mathbb{D}_\pm$ and their complex conjugates $\bar{\mathbb{D}}_\pm$ in terms of real $N=(1,1)$ derivatives $D_\pm$ and the generators $Q_\pm$ of the nonmanifest supersymmetries,

$$\mathbb{D}_\pm = \frac{1}{2}(D_\pm - iQ_\pm) \ , \quad \bar{\mathbb{D}}_\pm = \frac{1}{2}(D_\pm + iQ_\pm) \, , \tag{3.1}$$

and $N=(1,1)$ components of an unconstrained superfield Ψ as $\Psi| = \phi$, $Q_\pm\Psi| = \psi_\pm$, and $Q_+Q_-\Psi| = F$.

3.1 The semichiral vector multiplet

We first identify the $N=(1,1)$ components of the semichiral vector multiplet, and then describe various couplings to matter.

3.1.1 $N=(1,1)$ components of the gauge multiplet

We can find all the $N=(1,1)$ components of the semichiral gauge multiplet from the field strengths (2.16) except for the spinor connections $\Gamma_\pm$. The only linear combination of the gauge parameters Λ_R, Λ_L that does not enter algebraically in (2.12) is $(\Lambda_L+\bar{\Lambda}_L-\Lambda_R-\bar{\Lambda}_R)$, and hence the connections must transform as:

$$\delta_g\Gamma_\pm = \frac{1}{4}D_\pm(\Lambda_L+\bar{\Lambda}_L-\Lambda_R-\bar{\Lambda}_R)\Big| \ . \tag{3.2}$$

This allows us to determine the connections as:

$$\Gamma_+ = \left(\frac{1}{2}Q_+\mathbb{V}^L-\frac{1}{4}D_+\mathbb{V}'\right)\Big| \ , \quad \Gamma_- = -\left(\frac{1}{2}Q_-\mathbb{V}^R-\frac{1}{4}D_-\mathbb{V}'\right)\Big| \ , \tag{3.3}$$

where the $D_\pm$ terms vanish in Wess-Zumino gauge. The gauge-invariant component fields are just the projections of the $N=(2,2)$ field-strengths (2.16) and the field-strength of the connection $\Gamma_\pm$:

$$f = i(D_+\Gamma_- + D_-\Gamma_+) \ . \tag{3.4}$$

These are not all independent-they obey the Bianchi identity:

$$f = i\left(\mathbb{F}-\bar{\mathbb{F}}+\tilde{\mathbb{F}}-\bar{\tilde{\mathbb{F}}}\right)\Big| \ . \tag{3.5}$$

Thus this gauge multiplet is described by an $N=(1,1)$ gauge multiplet and three real unconstrained $N=(1,1)$ scalar superfields:

$$\hat{d}^1 = \left(\mathbb{F}+\bar{\mathbb{F}}\right)\Big| \ , \quad \hat{d}^2 = \left(\tilde{\mathbb{F}}+\bar{\tilde{\mathbb{F}}}\right)\Big| \ , \quad \hat{d}^3 = i\left(\mathbb{F}-\bar{\mathbb{F}}-\tilde{\mathbb{F}}+\bar{\tilde{\mathbb{F}}}\right)\Big| \ . \tag{3.6}$$

Though not essential, the simplest way to find the $N=(1,1)$ reduction of various $N=(2,2)$ quantities is to go to a Wess-Zumino gauge, that is reducing the $N=(2,2)$ gauge parameters to a *single* $N=(1,1)$ gauge parameter by gauging away all $N=(1,1)$ components with algebraic gauge transformations. Here this means imposing

$$\begin{array}{lll} \mathbb{V}^L| = 0 \ , & (Q_+\mathbb{V}^L)| = 2\Gamma_+ \ , & (Q_-\mathbb{V}^L)| = 0 \ , \\ \mathbb{V}^R| = 0 \ , & (Q_+\mathbb{V}^R)| = 0 \ , & (Q_-\mathbb{V}^R)| = -2\Gamma_- \ , \\ \mathbb{V}'| = 0 \ , & (Q_+\mathbb{V}')| = 0 \ , & (Q_-\mathbb{V}')| = 0 \ , \end{array} \tag{3.7}$$

on the gauge multiplet and

$$\Lambda^L| = \bar{\Lambda}^L| = -\Lambda^R| = -\bar{\Lambda}^R| \ , \quad (Q_-\Lambda^L)| = (Q_-\bar{\Lambda}^L)| = (Q_+\Lambda^R)| = (Q_+\bar{\Lambda}^R)| = 0 \tag{3.8}$$

on the gauge parameters. This leads directly to:

$$(Q_+Q_-\mathbb{V}^L)| = 2i(\hat{d}^1-\hat{d}^2) \ , \quad (Q_+Q_-\mathbb{V}^R)| = 2i(\hat{d}^1+\hat{d}^2) \ , \quad (Q_+Q_-\mathbb{V}')| = 2i\hat{d}^3 \ . \tag{3.9}$$

3.1.2 Coupling to matter

We start from the gauged $N=(2,2)$ Lagrange density:

$$K_{\mathbb{X}} = K_{\mathbb{X}}\left(\mathbb{X}_L + \bar{\mathbb{X}}_L + \mathbb{V}^L, \mathbb{X}_R + \bar{\mathbb{X}}_R + \mathbb{V}^R, i(\mathbb{X}_L - \bar{\mathbb{X}}_L + \mathbb{X}_R - \bar{\mathbb{X}}_R) + \mathbb{V}'\right) \ . \tag{3.10}$$

In the Wess-Zumino gauge defined above, we have

$$X_{L(R)} = \mathbb{X}_{L(R)}| \,, \tag{3.11}$$

and $N=(1,1)$ spinor components:

$$\begin{aligned} (Q_+\mathbb{X}_L)| &= iD_+X_L + \Gamma_+ \quad , \quad (Q_-\mathbb{X}_L)| = \psi_- \,, \\ (Q_-\mathbb{X}_R)| &= iD_-X_R - \Gamma_- \quad , \quad (Q_+\mathbb{X}_R)| = \psi_+ \,. \end{aligned} \tag{3.12}$$

Then for the tuple X^i and the isometry vector k^i defined as

$$\begin{aligned} k^i &\equiv k_{LR} = (i,-i,-i,i)\,, \\ X^i &= (X_L, \bar{X}_L, X_R, \bar{X}_R)\,, \end{aligned} \tag{3.13}$$

we write the gauge covariant derivative as it appears in [9]

$$\nabla_\pm X^i = D_\pm X^i - \Gamma_\pm k^i. \tag{3.14}$$

We can compute

$$\begin{aligned} (Q_+Q_-\mathbb{X}_L)| &= iD_+\psi_- + i(\hat{d}^1 - \hat{d}^2) + \hat{d}^3 \\ (Q_+Q_-\mathbb{X}_R)| &= -iD_+\psi_- + i(\hat{d}^1 + \hat{d}^2) + \hat{d}^3 \ . \end{aligned} \tag{3.15}$$

Using

$$\frac{\partial^2 K}{\partial X^i \partial X^j} k^i = 0 \quad \Rightarrow \quad \frac{\partial^2 K}{\partial X^i \partial X^j} D_\pm X^i = \frac{\partial^2 K}{\partial X^i \partial X^j} \nabla_\pm X^i \,, \tag{3.16}$$

we obtain the gauged $N=(1,1)$ Lagrange density

$$E_{ij}\nabla_+X^i\nabla_-X^j + K_i L^i{}_\alpha \hat{d}^\alpha \,, \tag{3.17}$$

with:

$$L = \begin{pmatrix} i & -i & 1 \\ -i & i & 1 \\ i & i & 1 \\ -i & -i & 1 \end{pmatrix} \ . \tag{3.18}$$

Here $E = \frac{1}{2}(g+B)$ in the reduced Lagrange density is that same as for the ungauged σ-model [14, 18].

3.1.3 The vector multiplet action

Introducing the notation

$$\mathbb{F}^i \equiv (\mathbb{F}, \bar{\mathbb{F}}, \tilde{\mathbb{F}}, \bar{\tilde{\mathbb{F}}}) \ , \quad d^i \equiv (f, \hat{d}^1, \hat{d}^2, \hat{d}^3) \, , \tag{3.19}$$

and using the (twisted)chirality properties

$$\bar{\mathbb{D}}_\pm \mathbb{F} = \mathbb{D}_\pm \bar{\mathbb{F}} = \bar{\mathbb{D}}_+ \tilde{\mathbb{F}} = \mathbb{D}_- \tilde{\mathbb{F}} = \mathbb{D}_+ \bar{\tilde{\mathbb{F}}} = \bar{\mathbb{D}}_- \bar{\tilde{\mathbb{F}}} = 0 \, , \tag{3.20}$$

we find

$$(Q_\pm \mathbb{F}^i)| = J_\pm{}^i{}_j \, M^j{}_k (D_\pm \hat{d}^k) \, , \tag{3.21}$$

with

$$M = \frac{1}{4} \begin{pmatrix} -i & 2 & 0 & -i \\ i & 2 & 0 & i \\ -i & 0 & 2 & i \\ i & 0 & 2 & -i \end{pmatrix} \ , \quad J_\pm \equiv \mathrm{diag}(i, -i, \pm i, \mp i) \ . \tag{3.22}$$

Starting from an $N=(2,2)$ action:

$$S_{\mathbb{X}} = \int d^2\xi \, D_+ D_- Q_+ Q_- \left(a \, \mathbb{F}\bar{\mathbb{F}} - b \, \tilde{\mathbb{F}} \bar{\tilde{\mathbb{F}}} \right) \tag{3.23}$$

we write the reduction to $N=(1,1)$ in terms of the gauge-invariant $N=(1,1)$ components $\hat{d}^i$:

$$S_{\mathbb{X}} = \frac{1}{2} \int d^2\xi \, D_+ D_- \left(D_+ \hat{d}^i \, D_- \hat{d}^j \, g_{ij} \right) \, , \tag{3.24}$$

where

$$g = \frac{1}{8} \begin{pmatrix} a+b & 0 & 0 & a-b \\ 0 & 4a & 0 & 0 \\ 0 & 0 & 4b & 0 \\ a-b & 0 & 0 & a+b \end{pmatrix} \ . \tag{3.25}$$

To obtain real and positive definite g we require $ab > 0$ which yields one $N=(1,1)$ gauge multiplet and three scalar multiplets. In particular, when $a = b$, we find the usual diagonal action.

Other gauge-invariant terms are possible; these are general superpotentials and have the form

$$S_P = \int i\mathbb{D}_+ \mathbb{D}_- \, P_1(\mathbb{F}) + \int i\bar{\mathbb{D}}_+ \bar{\mathbb{D}}_- \, \bar{P}_1(\bar{\mathbb{F}}) + \int i\mathbb{D}_+ \bar{\mathbb{D}}_- \, P_2(\tilde{\mathbb{F}}) + \int i\bar{\mathbb{D}}_+ \mathbb{D}_- \, \bar{P}_2(\bar{\tilde{\mathbb{F}}}) \, , \tag{3.26}$$

where P are holomorphic functions. These terms reduce trivially to give:

$$S_P = 2 \int i D_+ D_- \mathrm{Re} \left(P_1 \left(\frac{1}{2} \hat{d}^1 - \frac{i}{4} (f + \hat{d}^3) \right) + P_2 \left(\frac{1}{2} \hat{d}^2 - \frac{i}{4} (f - \hat{d}^3) \right) \right) . \tag{3.27}$$

Particular examples of such superpotentials include mass and Fayet-Iliopoulos terms.

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3.1.4 Linear terms

To perform T-duality transformations, one gauges an isometry, and then constrains the field-strength to vanish [9, 13]. We will discuss T-duality for generalized Kähler geometry in detail in [10]; it was introduced (without exploring the gauge aspects) in [11, 12]. Here we describe the $N=(2,2)$ superspace coupling and its reduction to $N=(1,1)$. We constrain the field-strengths to vanish using unconstrained complex Lagrange multiplier superfields $\Psi, \tilde{\Psi}$

$$\mathcal{L}_{\text{linear}} = \Psi\mathbb{F} + \bar{\Psi}\bar{\mathbb{F}} + \tilde{\Psi}\tilde{\mathbb{F}} + \bar{\tilde{\Psi}}\bar{\tilde{\mathbb{F}}} \; ; \tag{3.28}$$

integrating by parts, we can re-express this in terms of chiral and twisted chiral Lagrange multipliers $\phi = \bar{\mathbb{D}}_+\bar{\mathbb{D}}_-\Psi$, $\chi = \bar{\mathbb{D}}_+\mathbb{D}_-\tilde{\Psi}$ to obtain

$$\mathcal{L}_{\text{linear}} = \phi\mathbb{V} + \bar{\phi}\bar{\mathbb{V}} + \chi\tilde{\mathbb{V}} + \bar{\chi}\bar{\tilde{\mathbb{V}}}. \tag{3.29}$$

This reduces to an $N=(1,1)$ superspace Lagrange density (up to total derivative terms)

$$\begin{aligned}\mathcal{L}_{\text{linear}} = \; & \phi(i\hat{d}^3 - 2\hat{d}^1 + if) + \bar{\phi}(i\hat{d}^3 + 2\hat{d}^1 + if) \\ & + \chi(i\hat{d}^3 + 2\hat{d}^2 - if) + \bar{\chi}(i\hat{d}^3 - 2\hat{d}^2 - if)\,,\end{aligned} \tag{3.30}$$

where $\phi, \bar{\phi}, \chi, \bar{\chi}$ are the obvious $N=(1,1)$ projections of the corresponding $N=(2,2)$ Lagrange multipliers. When we perform a T-duality transformation, we add this to the Lagrange density (3.17).

3.2 The large vector multiplet

We now study the $N=(1,1)$ components of the large vector multiplet.

3.2.1 $N=(1,1)$ gauge invariants

Starting with the eight $N=(2,2)$ second-order gauge invariants (2.19), we descend to $N=(1,1)$ superspace and identify the $N=(1,1)$ gauge field-strength.

Imposing the condition that the $N=(1,1)$ gauge connection transforms as

$$\delta_g A_\pm = \frac{1}{4} D_\pm(\bar{\tilde{\Lambda}} + \tilde{\Lambda} - \bar{\Lambda} - \Lambda)\,, \tag{3.31}$$

we find the quantities

$$\begin{aligned}A_+ &= -\left(\frac{1}{4}Q_+(V^\phi - V^\chi)\right)\Big| = \left(\frac{i}{4}Q_+(\tilde{V} - \bar{\tilde{V}})\right)\Big|\,, \\ A_- &= -\left(\frac{1}{4}Q_-(V^\phi + V^\chi)\right)\Big| = \left(\frac{i}{4}Q_-(V - \bar{V})\right)\Big|\,;\end{aligned} \tag{3.32}$$

of course, any gauge-invariant spinor may be added to $A_\pm$. It is useful to introduce the real and imaginary parts of $\mathbb{G}_\pm$:

$$\Xi^A_\pm = (\; \text{Re}(\mathbb{G}_\pm)|\,, \text{Im}(\mathbb{G}_\pm)|\;)\;. \tag{3.33}$$

These form a basis for the $N=(1,1)$ gauge-invariant spinors. The field-strength of the connection $A_\pm$

$$f = i(D_+A_- + D_-A_+) = i(Q_+\Xi^2_- + Q_-\Xi^2_+) \tag{3.34}$$

is manifestly gauge invariant. The remaining $N=(1,1)$ gauge-invariant scalars are:

$$\begin{aligned}
\hat{q}^1 &= i(Q_-\Xi^1_+ - Q_+\Xi^1_-)\,,\\
\hat{q}^2 &= i(Q_-\Xi^1_+ + Q_+\Xi^1_-)\,,\\
\hat{q}^3 &= i(Q_-\Xi^2_+ - Q_+\Xi^2_-)\ .
\end{aligned} \tag{3.35}$$

The decomposition of the $N=(2,2)$ invariants W, B is

$$F^i = \left.\begin{pmatrix} W\\ B\\ \bar{W}\\ \bar{B}\\ \tilde{W}\\ \tilde{B}\\ \bar{\tilde{W}}\\ \bar{\tilde{B}}\end{pmatrix}\right| = \frac{1}{2}\begin{pmatrix}
-i & -i & 1 & 1 & 0 & 1 & 0 & i\\
i & -i & -1 & 1 & 1 & 0 & i & 0\\
i & i & 1 & 1 & 0 & 1 & 0 & -i\\
-i & i & -1 & 1 & 1 & 0 & -i & 0\\
-i & -i & -1 & 1 & -1 & 0 & 0 & -i\\
i & -i & 1 & 1 & 0 & -1 & -i & 0\\
i & i & -1 & 1 & -1 & 0 & 0 & i\\
-i & i & 1 & 1 & 0 & -1 & i & 0
\end{pmatrix}\begin{pmatrix} iD_+\Xi^1_-\\ iD_-\Xi^1_+\\ iD_+\Xi^2_-\\ iD_-\Xi^2_+\\ \hat{q}^1\\ \hat{q}^2\\ \hat{q}^3\\ f\end{pmatrix}\ . \tag{3.36}$$

3.2.2 Matter couplings in $N=(1,1)$ superspace

We start from the gauged $N=(2,2)$ Lagrange density:

$$K_\phi\left(\phi+\bar{\phi}+V^\phi, \chi+\bar{\chi}+V^\chi, i(\phi-\bar{\phi}+\chi-\bar{\chi})+V'\right)\ . \tag{3.37}$$

We reduce to $N=(1,1)$ superfields, which in the Wess-Zumino gauge

$$V^\phi| = 0\ ,\quad V^\chi| = 0\ ,\quad V'| = 0\ , \tag{3.38}$$

are simply

$$\begin{aligned}
\phi| &= \phi\,,\\
\chi| &= \chi\,,\\
(Q_+\phi)| &= +iD_+\phi - (\Xi^1_+ + i\Xi^2_+) - A_+\,,\\
(Q_+\chi)| &= +iD_+\chi - (\Xi^1_+ + i\Xi^2_+) + A_+\,,\\
(Q_-\phi)| &= +iD_-\phi - (\Xi^1_- + i\Xi^2_-) - A_-\,,\\
(Q_-\chi)| &= -iD_-\chi + (\Xi^1_- - i\Xi^2_-) - A_-\ .
\end{aligned} \tag{3.39}$$

It is useful to introduce the notation

$$\varphi^i = (\phi, \bar{\phi}, \chi, \bar{\chi}) \tag{3.40}$$

and the covariant derivatives

$$\nabla_\pm\varphi^i = D_\pm\varphi^i + A_\pm k^i\ . \tag{3.41}$$

This gives

$$Q_\pm\varphi^i = J_\pm{}^i{}_j\nabla_\pm\varphi^j + \Xi^1_\pm J_\mp{}^i{}_j k^j + \Xi^2_\pm \Pi^i{}_j k^j \tag{3.42}$$

and

$$\begin{aligned} 2Q_+Q_-\varphi^i \;=\; & D_+(\Pi^i{}_j\nabla_-\varphi^j - \Xi^1_- k^i - 2\Xi^2_- J_-{}^i{}_j k^j) \\ & -D_-(\Pi^i{}_j\nabla_+\varphi^j - \Xi^1_+ k^i - 2\Xi^2_+ J_+{}^i{}_j k^j) + 2\tilde{L}^i{}_\alpha \hat{q}^\alpha \end{aligned} \tag{3.43}$$

where $\alpha = 1,2,3$ and

$$\tilde{L} = -\frac{i}{2}\begin{pmatrix} 2 & 0 & i \\ 2 & 0 & -i \\ 0 & 2 & i \\ 0 & 2 & -i \end{pmatrix} \tag{3.44}$$

The $N=(1,1)$ superspace Lagrange density is (after integrating by parts and using the isometry)

$$\begin{aligned} \mathcal{L} = K_{ij}\Bigg[& -\tfrac{1}{2}\left(\nabla_+\varphi^i\left(\Pi^j{}_l\nabla_-\varphi^l - 2\Xi^2_- J_-{}^j{}_l k^l\right) + \left(\Pi^i{}_k\nabla_+\varphi^k - 2\Xi^2_+ J_+{}^i{}_k k^k\right)\nabla_-\varphi^j\right) \\ & + \left(J_+{}^i{}_k\nabla_+\varphi^k + \Xi^1_+ J_-{}^i{}_k k^k + \Xi^2_+\Pi^i{}_k k^k\right)\left(J_-{}^j{}_l\nabla_-\varphi^l + \Xi^1_- J_+{}^j{}_l k^l + \Xi^2_-\Pi^j{}_l k^l\right)\Bigg] \\ & + K_i\tilde{L}^i{}_\alpha\hat{q}^\alpha \ . \end{aligned} \tag{3.45}$$

The large vector multiplet has the gauge-invariant spinors $\Xi^A_\pm$; it is useful to isolate their contribution to expose the underlying $N=(1,1)$ gauged nonlinear σ-model. We define the matrices:

$$E_{kl} = \frac{1}{2}K_{ij}\left(2J_+{}^i{}_k J_-{}^j{}_l - \Pi^i{}_k\delta^j{}_l - \Pi^j{}_l\delta^i{}_k\right) \tag{3.46}$$

$$E_{Al} = \begin{pmatrix} K_{ij}J_-{}^i{}_k k^k J_-{}^j{}_l \\ K_{ij}\left(J_+{}^i{}_k k^k\delta^j{}_l + \Pi^i{}_k k^k J_-{}^j{}_l\right) \end{pmatrix} \tag{3.47}$$

$$E_{kA} = \left(K_{ij}J_+{}^i{}_k J_+{}^j{}_l k^l\,,\ K_{ij}\Big(J_-{}^j{}_l k^l\delta^i{}_k + J_+{}^i{}_k\Pi^j{}_l k^l\Big)\right) \tag{3.48}$$

$$E_{AB} = \begin{pmatrix} K_{ij}J_-{}^i{}_k k^k J_+{}^j{}_l k^l & K_{ij}\Pi^i{}_k k^k J_+{}^j{}_l k^l \\ K_{ij}J_-{}^i{}_k k^k\Pi^j{}_l k^l & K_{ij}\Pi^i{}_k k^k\Pi^j{}_l k^l \end{pmatrix} \tag{3.49}$$

We find

$$\begin{aligned} \mathcal{L} = & \left(\Xi^A_+ + \nabla_+\varphi^i E_{iC}E^{CA}\right)E_{AB}\left(\Xi^B_- + E^{BD}E_{Dj}\nabla_-\varphi^j\right) \\ & +\nabla_+\varphi^i\left(E_{ij} - E_{iA}E^{AB}E_{Bj}\right)\nabla_-\varphi^j + K_i\tilde{L}^i{}_\alpha\hat{q}^\alpha \end{aligned} \tag{3.50}$$

with E^{AB} the inverse of E_{AB}.

3.2.3 The vector multiplet action

A general $N=(2,2)$ action for the large multiplet can be written as

$$S_a = \int d^2\xi D_+D_-Q_+Q_-\left(F^iF^j g_{ij} + \mathbb{G}^A_+\mathbb{G}^B_- m_{AB}\right)\,, \tag{3.51}$$

where the ranges for indices are $i, j = 1, \ldots, 8$; $AB = 1, 2$, and the spinor invariants were arranged into tuples

$$\mathbb{G}^A_\pm = (\mathbb{G}_\pm, \bar{\mathbb{G}}_\pm) \ . \tag{3.52}$$

Other terms of the type $(\mathbb{D}_\pm, \bar{\mathbb{D}}_\pm)(\mathbb{G}_\pm, \bar{\mathbb{G}}_\pm)$ could be integrated by parts to give the W and B invariants. One could also add superpotential terms.

This action can be reduced to $N = (1, 1)$ using the block-(twisted)chirality of F and the semichirality of $\mathbb{G}$. In general, one finds terms with higher derivatives; it does not seem possible to find a sensible kinetic action, but we leave a complete analysis for future work.

3.2.4 Linear terms

As discussed above for the semichiral vector multiplet, linear couplings of unconstrained Lagrange multiplier fields multiplying the field-strengths are needed to discuss T-duality. In $N = (2, 2)$ superspace, we constrain the field-strengths $\mathbb{G}_\pm$ to vanish with unconstrained complex spinor Lagrange multiplier superfields $\Psi_\mp$:

$$\mathcal{L}_{\text{linear}} = i \left(\Psi_+ \mathbb{G}_- + \Psi_- \mathbb{G}_+ + \bar{\Psi}_+ \bar{\mathbb{G}}_- + \bar{\Psi}_- \bar{\mathbb{G}}_- \right) \ . \tag{3.53}$$

When we integrate by parts and define semichiral Lagrange multpliers $\mathbb{X}_{L,R} = -i\bar{\mathbb{D}}_\pm \Psi_\mp$, we find

$$\mathcal{L}_{\text{linear}} = \mathbb{X}_L V + \bar{\mathbb{X}}_L \bar{V} + \mathbb{X}_R \tilde{V} + \bar{\mathbb{X}}_R \bar{\tilde{V}} \ . \tag{3.54}$$

Reducing to $N = (1, 1)$ superspace, and defining $N = (1, 1)$-components for the Lagrange multipliers as in (3.11), (3.12) we find

$$\begin{aligned} \mathcal{L}_{\text{linear}} = {} & \psi_- \left(i\Xi^1_+ - \Xi^2_+ \right) + \frac{1}{2} X_L \left((\hat{q}^2 + \hat{q}^1) + i(f + \hat{q}^3) \right) \\ & + \bar{\psi}_- \left(-i\Xi^1_+ - \Xi^2_+ \right) + \frac{1}{2} \bar{X}_L \left(-(\hat{q}^2 + \hat{q}^1) + i(f + \hat{q}^3) \right) \\ & + \psi_+ \left(-i\Xi^1_- + \Xi^2_- \right) + \frac{1}{2} X_R \left(-(\hat{q}^2 - \hat{q}^1) - i(f - \hat{q}^1) \right) \\ & + \bar{\psi}_+ \left(i\Xi^1_+ - i\Xi^2_+ \right) + \frac{1}{2} \bar{X}_R \left((\hat{q}^2 - \hat{q}^1) - i(f - \hat{q}^1) \right) \ . \end{aligned} \tag{3.55}$$

We can easily integrate out $\psi_\pm$ and their complex conjugates; this eliminates $\Xi^A_\pm$ from the action. We are then left with the usual T-duality transformation as we shall discuss in [10].

Note added. As we were completing our work, we became aware of related work by S.J. Gates and W. Merrell; we thank them for agreeing to delay their work and post simultaneously.

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